

Adaptively Secure Non-Interactive Threshold Cryptosystems: New Framework and Constructions

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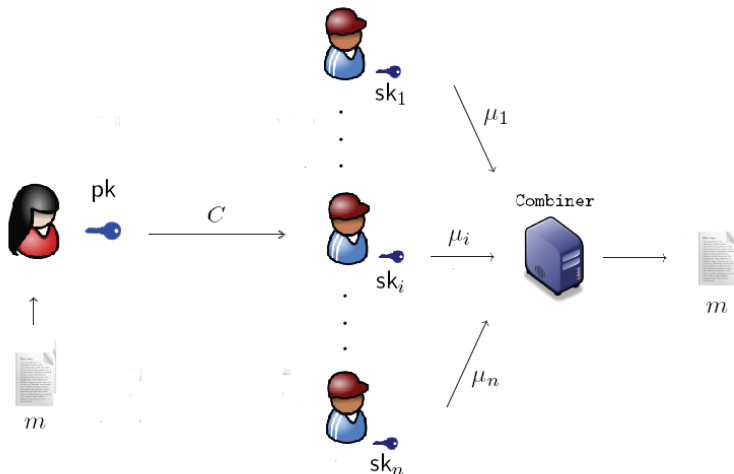
Darmstadt

Threshold Cryptography

- Introduced by Desmedt-Frankel (Crypto'89) and Boyd (IMA'89)
- Split private keys into n shares SK_1, \dots, SK_n so that knowing strictly less than $t \leq n$ shares is useless to the adversary.
- At least $t \leq n$ shareholders must contribute to private key operations.
 - Decryption requires the cooperation of t decryption servers.
 - Signing requires at least t servers to run a joint signing protocol.
- *Robustness*: up to $t - 1 \leq n$ malicious servers cannot prevent an honest majority from decrypting/signing.

Threshold Cryptography

The public-key encryption case:



Outline

1 Threshold Cryptography

- Static vs Adaptive corruptions
- Security notions: CCA2 security and consistency

2 A New Framework for Non-Interactive Threshold CCA2 Encryption

- All-But-One Perfectly Sound Hash Proof Systems
- General Construction
- Instantiations based on Simple Assumptions
- Efficiency comparisons

Static vs Adaptive corruptions

- Static corruptions: adversary corrupts servers *before* seeing the public key.

Robust threshold cryptosystems with IND-CCA2 security:

- Shoup-Gennaro (Eurocrypt'98): in the ROM.
- Canetti-Goldwasser (Eurocrypt'99): interactive decryption or storage of many pre-shared secrets; non-optimal resilience $t \approx n/3$.
- Abe (Crypto'99): optimal-resilience $t = (n - 1)/2$ in [CG'99].
- Dodis-Katz (TCC'05): generic constructions; ciphertexts of size $O(n)$.
- Boneh-Boyen-Halevi (CT-RSA'06): no interaction needed for robustness.
- Wee (Eurocrypt'11): generic constructions from (threshold) extractable hash proof systems.

Static vs Adaptive corruptions

- Adaptive corruptions: adversary corrupts up to $t - 1$ servers *at any time*.
 - Canetti *et al.* (Crypto'99) and Frankel-MacKenzie-Yung (ESA'99, Asiacrypt'99): reliance on erasures.
 - Jarecki-Lysyanskaya (Eurocrypt'00): no need for erasures, but interaction required at decryption with Cramer-Shoup.
 - Lysyanskaya-Peikert (Asiacrypt'01): adaptively secure signatures with interaction.
 - Abe-Fehr (Crypto'04): adaptively secure UC-secure threshold signatures and encryption with interaction.
 - Almansa-Damgaard-Nielsen (Eurocrypt'06): adaptively secure proactive RSA signatures.

Threshold Cryptosystems: Our Goal

- Until recently (and despite more than 10 years of research), adaptive security has not been achieved in threshold encryption schemes with:
 - CCA2-security
 - Non-interactive schemes
 - Robustness against malicious adversaries
 - Optimal resilience ($t = (n - 1)/2$)
 - No erasures for shareholders
 - Share size independent of t, n
 - Proof in the standard model

CCA2-Secure Non-interactive Threshold Encryption

Recently (ICALP'11), we described:

- The first *adaptively* secure *fully non-interactive* threshold cryptosystem with
 - CCA2 security and robustness w/o random oracles
 - Short (*i.e.*, $O(1)$ -size) private key shares
- The construction
 - Builds on the dual system encryption approach (Waters, Crypto'09) and the Lewko-Waters techniques (TCC'10).
 - Handles adaptive corruptions by instantiating Boneh-Boyen-Halevi (CT-RSA'06) in bilinear groups of order $N = p_1 p_2 p_3$.
 - ⇒ Ciphertexts live in the subgroup \mathbb{G}_{p_1} , private keys in $\mathbb{G}_{p_1 p_3}$
- Gives adaptively secure non-interactive threshold signatures; also yields non-interactive *forward-secure* threshold encryption.

CCA2-Secure Non-interactive Threshold Encryption

New results: a new approach from hash proof systems with *public* verifiability

- Combines universal hash proofs with simulation-sound proofs of ciphertext validity (\Rightarrow publicly verifiable ciphertexts).
- Proofs of validity associated with tags and perfectly sound on *all but one* tag.
- New constructions in groups of order $N = p_1 p_2$ and *prime-order* groups
 - Better efficiency
 - Tighter security (no gap $O(q)$ in the reduction) under a *single* assumption
 - Easier to combine with a DKG protocol

Security of Non-interactive Threshold Encryption

- Chosen-ciphertext (IND-CCA) security:
 1. Challenger generates PK , $SK = (SK_1, \dots, SK_n)$ and gives PK to \mathcal{A} .
 2. \mathcal{A} makes adaptive queries
 - Corruption $i \in \{1, \dots, n\}$: \mathcal{A} receives SK_i (up to $t - 1$ queries allowed).
 - Decryption (i, C) : \mathcal{A} receives $\mu_i = \text{Share-Decrypt}(PK, i, SK_i, C)$
 3. \mathcal{A} chooses M_0, M_1 and gets $C^* = \text{Encrypt}(PK, M_\beta)$ for some $\beta \xleftarrow{R} \{0, 1\}$.
 4. \mathcal{A} makes further queries with restrictions.
 5. \mathcal{A} outputs $\beta' \in \{0, 1\}$ and wins if $\beta' = \beta$

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Security of Non-interactive Threshold Encryption

- Consistency:

1. Challenger generates PK , $SK = (SK_1, \dots, SK_n)$ and gives PK to \mathcal{A} .
2. \mathcal{A} makes adaptive queries
 - Corruption query $i \in \{1, \dots, n\}$: \mathcal{A} receives SK_i .
 - Decryption query (i, C) : \mathcal{A} receives $\mu_i = \text{Share-Decrypt}(PK, i, SK_i, C)$
3. \mathcal{A} outputs a ciphertext C and sets $S = \{\mu_1, \dots, \mu_t\}$, $S' = \{\mu'_1, \dots, \mu'_t\}$ of shares such that
 - C is a valid ciphertext.
 - S and S' are sets of valid shares.
 - $\text{Combine}(PK, C, S) \neq \text{Combine}(PK, C, S')$.

A New Framework for Adaptive Security

Based on Hash Proof Systems:

- Let \mathcal{C} be a set and $\mathcal{V} \subset \mathcal{C}$ be a subset; let (pk, sk) be a key pair such that
 - If $\Phi \in \mathcal{V}$, $\text{PrivEval}(sk, \Phi)$ is completely fixed by Φ and pk (and computable as $\text{PubEval}(pk, \Phi, r)$ using a witness r that $\Phi \in \mathcal{V}$).
 - If $\Phi \in \mathcal{C} \setminus \mathcal{V}$, $\text{PrivEval}(sk, \Phi)$ is information-theoretically hidden.
- $D_1 = \{\Phi \mid \Phi \stackrel{r}{\leftarrow} \mathcal{V}\}$ is indistinguishable from $D_0 = \{\Phi \mid \Phi \stackrel{r}{\leftarrow} \mathcal{C} \setminus \mathcal{V}\}$.
- Message M can be encrypted as $(C_0, C_1) = (M \cdot \text{PubEval}(pk, \Phi, r), \Phi)$ and decrypted as $M = C_0 \cdot \text{PrivEval}(sk, C_1)^{-1}$.
- In the security proof, to decide if $\Phi^* \in \mathcal{V}$ or $\Phi^* \in \mathcal{C} \setminus \mathcal{V}$, set

$$(C_0^*, C_1^*) = (M_\beta \cdot \text{PrivEval}(sk, \Phi^*), \Phi^*).$$

A New Framework for Adaptive Security

- In the security proof, to decide if $\Phi^* \in \mathcal{V}$, set

$$(C_0^*, C_1^*) = (M_\beta \cdot \text{PrivEval}(sk, \Phi^*), \Phi^*).$$

- Private key sk is available to the reduction.
- For CCA2-security, the reduction should reject $(C_0, C_1 = \Phi)$ if $\Phi \notin \mathcal{V}$.
 \Rightarrow Cramer-Shoup uses non-interactive designated-verifier proofs that $\Phi \in \mathcal{V}$
- In the threshold setting, $\Phi \in \mathcal{V}$ cannot be checked from partial decryptions.
 \Rightarrow Existing solutions [CG99, JL00, AF04] require interaction to render ciphertexts with $\Phi \notin \mathcal{V}$ harmless.

A New Framework for Adaptive Security

Our approach: *All-But-One Perfectly Sound* Hash Proof Systems

- Combination between
 - Universal hash proofs (simulator knows private keys in reduction).
 - Simulation-sound proofs of ciphertext validity (publicly verifiable ciphertexts).
- Proofs of validity associated with tags and perfectly sound on *all but one* tag.
- Gives new constructions
 - Based on the Subgroup Decision assumption in composite order groups with two primes $N = p_1 p_2$.
 - Or Groth-Sahai proofs (D-Linear/SXDH assumptions) in *prime-order* groups:
⇒ Better efficiency; easier to combine with a DKG protocol.

All-But-One Perfectly Sound Hash Proof Systems

Non-interactive proofs that $\Phi \in \mathcal{V}$ are associated with tags

- Two distinct setup procedures
 - $\text{SetupSound}(\lambda, t, n)$: gives $(pk, \{sk_i\}_{i=1}^n)$ where pk yields sound proofs.
 - $\text{SetupABO}(\lambda, t, n, \text{tag}^*)$: gives $(pk, \{sk_i\}_{i=1}^n)$ and a trapdoor τ such that proofs are perfectly sound on all tags but tag^* .
- Two distinct proving algorithms
 - $\text{Prove}(pk, \text{tag}, r, \Phi)$: returns real proofs using the witness r that $\Phi \in \mathcal{V}$.
 - $\text{SimProve}(pk, \tau, \text{tag}^*, \Phi)$: returns a simulated proof for any $\Phi \in \mathcal{C}$.

All-But-One Perfectly Sound Hash Proof Systems

Main properties:

- **SETUP INDISTINGUISHABILITY:** $\text{SetupSound}(\lambda, t, n)$ and $\text{SetupABO}(\lambda, t, n, \text{tag}^*)$ have indistinguishable public outputs.
- **ALL-BUT-ONE SOUNDNESS:**
 - a. For any $(pk, (sk_1, \dots, sk_n), \tau) \leftarrow \text{SetupABO}(\lambda, t, n, \text{tag}^*)$ and any $\text{tag} \neq \text{tag}^*$, if $\pi_{\mathcal{V}}$ is a valid proof w.r.t. tag , then $\Phi \in \mathcal{V}$.
 - b. For any $(pk, (sk_1, \dots, sk_n), \tau) \leftarrow \text{SetupABO}(\lambda, t, n, \text{tag}^*)$, $\text{SimProve}(pk, \tau, \text{tag}^*, \Phi)$ gives a NIZK proof that $\Phi \in \mathcal{V}$ for any $\Phi \in \mathcal{C}$.

General Construction of Threshold CCA2 Cryptosystem

- **Keygen** (λ, t, n) : runs $\text{SetupSound}(\lambda, t, n)$ to obtain $(pk, \{sk_i\}_{i=1}^n)$.
- **Encrypt** (pk, M) : generate a one-time signature key pair $(SK, VK) \leftarrow \mathcal{G}(\lambda)$,
 1. Sample $\Phi \xleftarrow{r} \mathcal{V}$ using random coins r .
 2. Compute $C_0 = M \cdot \text{PubEval}(pk, r, \Phi)$.
 3. Compute a proof $\pi_{\mathcal{V}} \leftarrow \text{Prove}(pk, VK, \Phi)$ that $\Phi \in \mathcal{V}$.

Return $C = (VK, C_0, \Phi, \pi_{\mathcal{V}}, \sigma)$, where $\sigma = \mathcal{S}(SK, (C_0, \Phi, \pi_{\mathcal{V}}))$.

- **Share-Decrypt** (sk_i, pk, C) :
 1. Return \perp if $\mathcal{V}(VK, \sigma, (C_0, \Phi, \pi_{\mathcal{V}})) = 0$ or $\pi_{\mathcal{V}}$ is an invalid proof w.r.t. VK .
 2. Otherwise, compute a share $\text{PrivEval}(sk_i, \Phi)$ with a proof of validity.
- **Combine**: verifies all decryption shares and combines them.

Theorem

The scheme is consistent and **IND-CCA2** under adaptive corruptions if

- Σ is a strong one-time signature.
- The ABO-PS-HPS is secure

Idea of the proof of IND-CCA security:

- CRS only allows NIZK proofs in the challenge ciphertext and only the challenger can generate *one* fake proof.
- Adversary can only prove true statements (cf. one time simulation-soundness).
- Simulator knows the decryption keys (as in HPS-based proofs).

Instantiation in groups of order $N = p_1 p_2$

Subgroup Decision Problem: in a group \mathbb{G} of order $N = p_1 p_2$, given $(g \in \mathbb{G}_{p_1}, h \in \mathbb{G})$ and η , decide if $\eta \in_R \mathbb{G}_{p_1}$ or $\eta \in_R \mathbb{G}$.

An ordinary Hash Proof System: let $\mathcal{C} = \mathbb{G}$ and $\mathcal{V} = \mathbb{G}_{p_1}$.

- **Setup(λ):**

1. Choose a group \mathbb{G} of order $N = p_1 p_2$ with $g \xleftarrow{R} \mathbb{G}_{p_1}$.
2. Set $X = g^x$ with $x \xleftarrow{R} \mathbb{Z}_N$.
3. Let $H : \mathbb{G} \rightarrow \{0, 1\}^\ell$ be a pairwise independent hash function for some ℓ .

Output $pk = (\mathbb{G}, N, g, X, H)$ and $sk = x$.

- **PubEval(pk, r, Φ):** given $r \in \mathbb{Z}_N$ such that $\Phi = g^r$, output $H(X^r)$.
- **PrivEval(sk, Φ):** given $\Phi \in \mathbb{G}_{p_1}$, output $H(\Phi^x)$.

Instantiation in groups of order $N = p_1 p_2$

Define $\mathcal{C} = \mathbb{G}$ and $\mathcal{V} = \mathbb{G}_{p_1}$.

- $\text{SetupSound}(\lambda, t, n)$: chooses $g \xleftarrow{R} \mathbb{G}_{p_1}$, $u, v \xleftarrow{R} \mathbb{G}$.
- $\text{SetupABO}(\lambda, t, n, \text{tag}^*)$: is like SetupSound but chooses $v = u^{-\text{tag}^*} \cdot g^\alpha$ where $\alpha \xleftarrow{R} \mathbb{Z}_N$ is the trapdoor $\tau := \alpha$.
- $\text{Prove}(pk, \text{tag}, r, \Phi)$: given $\Phi = g^r \in \mathbb{G}_{p_1}$, output $\pi_{\mathcal{V}} = (u^{\text{tag}} \cdot v)^r$ such that

$$e(g, \pi_{\mathcal{V}}) = e(\Phi, u^{\text{tag}} \cdot v),$$

which guarantees $\Phi \in \mathbb{G}_{p_1}$.

- $\text{SimProve}(pk, \tau, \text{tag}^*, \Phi)$: given $\tau = \alpha \in \mathbb{Z}_N$, output $\pi_{\mathcal{V}} = \Phi^\alpha$, which satisfies

$$e(g, \pi_{\mathcal{V}}) = e(\Phi, u^{\text{tag}^*} \cdot v)$$

since $u^{\text{tag}^*} \cdot v = g^\alpha$.

Instantiation in prime order groups

Instantiation based on Groth-Sahai proofs and the D-Linear assumption:

- **Linear Problem:** given $(g, g_1, g_2, g_1^a, g_2^b, Z)$, decide if $Z \stackrel{?}{=} g^{a+b}$.
- Equivalently, given

$$\vec{g}_1 = (g_1, 1, g), \quad \vec{g}_2 = (1, g_2, g), \quad \vec{\varphi} = (g_1^a, g_2^b, Z),$$

decide whether $\vec{g}_1, \vec{g}_2, \vec{\varphi}$ are linearly dependent (i.e., $\vec{\varphi} \stackrel{?}{=} \vec{g}_1^a \cdot \vec{g}_2^b$).

- To commit to $x \in \mathbb{Z}_p$, set $\vec{C} = \vec{\varphi}^x \cdot \vec{g}_1^{t_1} \cdot \vec{g}_2^{t_2}$.
- Dual mode commitments:
 - Perfect binding commitments and perfectly sound proofs if $\vec{\varphi} \notin \text{span}(\vec{g}_1, \vec{g}_2)$.
 - Perfectly hiding commitments and WI proofs if $\vec{\varphi} \in \text{span}(\vec{g}_1, \vec{g}_2)$.

Instantiation in prime order groups

Linear Problem: given $(g, g_1, g_2, g_1^a, g_2^b, Z)$, decide if $Z \stackrel{?}{=} g^{a+b}$.

An ordinary HPS: given $g_1, g_2, g \in \mathbb{G}$, let $\mathcal{C} = \mathbb{G}^3$ and $\mathcal{V} = (g_1^a, g_2^b, g^{a+b})$.

- **Setup**(λ): choose a group \mathbb{G} of order p with $g \stackrel{R}{\leftarrow} \mathbb{G}$ and set

$$pk = (\mathbb{G}, g, g_1, g_2, X_1 = g_1^{x_1} \cdot g^z, X_2 = g_2^{x_2} \cdot g^z)$$

where $sk = (x_1, x_2, z) \stackrel{R}{\leftarrow} \mathbb{Z}_p^3$.

- **PubEval**(pk, r, Φ): given $(r, s) \in \mathbb{Z}_p^2$ s.t. $(\Phi_1, \Phi_2, \Phi_3) = (g_1^r, g_2^s, g^{r+s})$, output

$$X_1^r \cdot X_2^s.$$

- **PrivEval**(sk, Φ): given $\Phi = (\Phi_1, \Phi_2, \Phi_3) \in \mathbb{G}^3$, output $\Phi_1^{x_1} \cdot \Phi_2^{x_2} \cdot \Phi_3^z$.

Instantiation in groups of prime order

Define $\mathcal{C} = \mathbb{G}^3$ and $\mathcal{V} = (g_1^a, g_2^b, g^{a+b})$.

- $\text{SetupSound}(\lambda, t, n)$: set

$$\vec{g}_1 = (g_1, 1, g), \quad \vec{g}_2 = (1, g_2, g), \quad \vec{\varphi} = \vec{g}_1^a \cdot \vec{g}_2^b.$$

- $\text{SetupABO}(\lambda, t, n, \text{tag}^*)$: is like $\text{SetupSound}(\lambda, t, n)$ but

$$\vec{\varphi} = \vec{g}_1^a \cdot \vec{g}_2^b \cdot (1, 1, g)^{-\text{tag}^*}$$

and the trapdoor is $\tau := (a, b) \in \mathbb{Z}_p^2$.

- $\text{Prove}(pk, \text{tag}, r, \Phi)$: given $\Phi = (\Phi_1, \Phi_2, \Phi_3) = (g_1^r, g_2^s, g^{r+s})$ and (r, s) , generate a proof that $\Phi \in \mathcal{V}$ w.r.t. the CRS $(\vec{g}_1, \vec{g}_2, \vec{\varphi} \cdot (1, 1, g)^{\text{tag}})$.
- $\text{SimProve}(pk, \tau, \text{tag}^*, \Phi)$: simulate a NIZK proof using $\tau = (a, b) \in \mathbb{Z}_p^2$ on the “fake” CRS $(\vec{g}_1, \vec{g}_2, \vec{\varphi} \cdot (1, 1, g)^{\text{tag}^*})$.

Efficiency comparisons

- Estimations at the 128-bit security level

Approaches	Group order	Assumptions	Ciphertext overhead (# of bits)
Dual system	$N = p_1 p_2 p_3 > 2^{3072}$	Subgroup Decision Assumptions	6144
ABO-PS-HPS	$p > 2^{512}$	D-Linear	10240
ABO-PS-HPS	$p > 2^{256}$	SXDH	3328

Figure: Comparisons in terms of ciphertext overhead

- Under D-Linear: 12 pairings to check ciphertexts (using batch-verification); sender computes 19 exponentiations.
- Under SXDH: only 6 pairings to check ciphertexts (with batch-verification); sender computes 7 exponentiations.

Conclusion

- We described
 - A framework for CCA2-secure *robust* and *non-interactive* threshold cryptosystems secure against *adaptive* corruptions
 - Constructions in prime order groups using simple assumptions
 - Better efficiency
 - Compatibility with adaptively secure DKG protocols
 - ... with tight security proofs using fewer assumptions
- Open problems:
 - Are there instantiations without pairings?
 - Can we do the same for threshold signatures?